# Math 3354

Syllabus:

- Online
- Webwork → Homework (20%)
  - → Quizzes (15%)
  - → Exams (70%)
- Webpage (video lectures, notes, etc)
- Covid 19

# Differential Equations

- Reality: modelling growth.
- Mathematics: Involving derivatives
  $$F(x, y, y', \dots y^{(n)}) = 0.$$
- Type: ODE (one variable)
  PDE (several variables)

<u>Ex</u>: $y'' - \cos x \, y' + y = 3x$ (ODE), $\frac{\partial y}{\partial x} = \frac{\partial y}{\partial z}$ (PDE)

- Order: Highest derivative

<u>Ex</u>: $y'' - \cos x \, y' + y = 3x \Rightarrow$ order 2

$y' - 2xy - y^2 = 0 \Rightarrow$ order 1

- Linearity: An n-th order ODE is linear if it is of the form

$$a_n(x) y^{(n)}(x) + a_{n-1}(x) y^{(n-1)}(x) + \dots + a_1(x) y' + a_0(x) y = g(x)$$

<u>Ex</u>: $y'' - \cos x \, y' + y = 3x$ : linear

$(y')^2 = x^2 - y^2$ : nonlinear

- Homogeneity: $g(x) \equiv 0.$

$y' - 2xy - y^2 = 0$ : homogeneous

$y'' - \cos x \, y' + y = 3x$ : non-homogeneous

- Solution : verify the equation.

<u>Ex</u> : $y' - 3y = 0$ (D.E.)

$$y = e^{3x}, \quad y' = 3e^{3x} = 3y, \quad y' - 3y = 0.$$

So $y = e^{3x}$ is a solution.

## 1.2 Initial Value Problems

<u>IVP</u> : Solving $F(x, y, \dots, y^{(n)}) = 0$

subject to
$$\begin{cases} y(x_0) = y_1 \\ y'(x_0) = y_2 \\ \vdots \\ y^{(n)}(x_0) = y_n. \end{cases}$$

<u>Ex</u> : $y' = 3x^2$ (D.E.), $y(2) = -1$ (IVP).

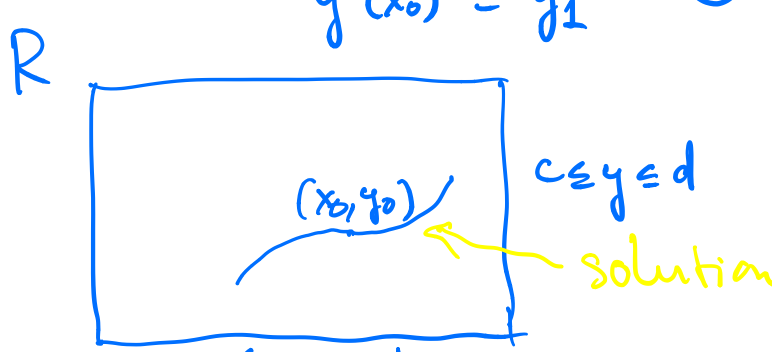
General sol : $y = x^3 + C$

$y' = 3x^2 + 0$, $y(2) = 2^3 + C = -1 \Rightarrow C = -9$

So $y = x^3 - 9$ is a sol. to IVP.

⊕ Fund. Existence & Uniqueness Theorem

Consider 1st order
$$\begin{cases} y' = f(x, y) \\ y(x_0) = y_1 \end{cases} \quad (*)$$



$a \le x \le b$

- Hypothesis: $f(x,y)$ and $\frac{\partial f}{\partial y}$ are continuous on R (rectangle)
- Conclusion: There is a solution!

Ex: $y' = -y^2$, $y(0) = 1$

$f(x,y) = -y^2$, $f_y = -2y$ are cont. in $R^2$

Thm $\Rightarrow$ IVP has a unique sol.

(f) Theorem for first order linear IVP.

$$a_1(x)\, y' + a_0(x)\, y = g(x), \quad y(x_0) = y_0.$$

$$\Leftrightarrow \quad y' = \frac{g(x) - a_0(x)y}{a_1(x)} = f(x,y)$$

- Hypothesis: $g(x)$, $a_0(x)$, $a_1(x)$ are continuous on some interval $I$,

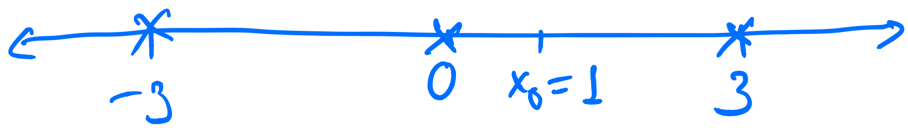
  $a_1(x) \neq 0$ on $I$
- Conclusion: There is a solution on $I$.

Ex: $(x^2-9)\, y' + x \cos x\, y = \frac{x+1}{x}$, $y(1) = 5$

$a_1(x) = x^2 - 9$, $a_0(x) = x\cos x$, $g(x) = \frac{x+1}{x}$.

- $a_1(x)$, $a_0(x)$ are cont. everywhere.
- $g(x)$ is cont as long as $x \neq 0$.
- $a_1(x) = 0$ at $x = \pm 3$

Real line




Maximal interval = largest interval containing $x_0$

$$= (0, 3).$$